Caution: This is practice for the final exam. *These are not the questions nor the physical systems on the final exam.*

Problem 1. Constrained system

Consider a point particle of mass m constrained to move on the *surface* of a sphere with no other forces applied.

(a) Find the appropriate generalized coordinates and write the Lagrangian.

(b) What are the conserved quantities?

(c) Without loss of generality, assume the angular momentum is along the z-axis. Solve the equations of motion. (*Note*: We can then use Euler angles to rotate this solution to get the most general solution.)

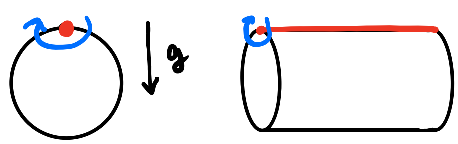
Problem 2. Motion of a rigid body

Consider a cylinder hanging from the wall, as shown in the picture. The cylinder is uniform, with radius R, length b, and total mass M. Gravity acts in the vertical direction. The cylinder oscillates around the equilibrium position. Ignore friction forces.

(a) Find the appropriate generalized coordinates and write the Lagrangian.

(b) Write the Hamiltonian and Hamilton's equations of motion.

(c) Compute the frequency of oscillation for small deviations from equilibrium.



Problem 3. Two particles

The most general quadratic potential for two particles constrained to one-dimension is $V(q_1, q_2) = \frac{1}{2}k_{11}q_1^2 + k_{12}q_1q_2 + \frac{1}{2}k_{22}q_2^2$.

(a) Assume both particles have mass m, write down the Lagrangian and derive the equations of motion.

(b) What are the normal frequencies?

(c) What conditions are needed on the k_{ij} to ensure stability?

Problem 4. Short answers

Give a brief sentence or calculation to the following

(a) A particle of charge q is moving in the presence of a cylinder of uniform, positive charge along the z-direction. What components of momentum and angular momentum are conserved?

(b) Confirm that the phase space transformation transformation $Q = \cos(\theta)q + \sin(\theta)p$, $P = -\sin(\theta)q + \cos(\theta)p$ is canonical.

(c) Give a brief statement of Noether's theorem.

(d) If a mass m is placed in a potential $V(x) = \frac{1}{3}x^3 - x$, what are the stationary points? Label them stable or unstable.